Worksheet 2nd

Topic : PERMUTATIONS

TIME : 4 X 45 minutes

STANDARD COMPETENCY :

1. To use the statistics rules, the rules of counting, and the characteristic of probability in problem solving.

BASIC COMPETENCY:

1.5 To use the rules of multiplication , permutation, and combination in problem solving.

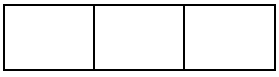
In this chapter, you will learn about :

- Permutation of r objects from n different objects.
- Permutations with Restrictions
- Permutation from n objects with a_1, a_2, a_3, … same objects.
- Circular Permutation

C. PERMUTATIONS

Recall Example 5:
The 6 possible arrangements of the 3 persons (A,B,C) are :

ABC ACB BAC BCA CAB CBA



These arrangements are also called **permutations.** A permutation is an arrangement of objects in a definite order.

Extending Example 5, the number of permutations of 9 different objects would be …x…x…x…x…x…x…x…x… = … !

Hence we have:

The number of permutations of n different objects is

$$n! = n(n-1)(n-2)\times\ldots\times 3\times 2\times 1$$

Example 8

Find the total number of different permutations of all the letters of the word PELUANG.

Solution

Notice that all the letters are *different*.

So the total number of permutations = … ! = …

Example 9

In how many ways can 9 different books be arranged on a shelf? If another book is added, what is the total number of permutations?

Solution

The number of ways = … !
With the addition of the 10th book, the total number of permutations = …! = …

C.1 Permutation of *r* objects from *n* different objects

Example 10

For the Dempo concert, 6 items are proposed. Only 4 items will put up at the concert. How many permutations of 4 concert items are there?

Solution

Let us use 4 boxes to represent the 4 concert items.

So the number of permutations is … x … x … x … = …
Using the factorial notation, we have ${}_6P_4$ = … x … x … x …

$$= \frac{\ldots\times\ldots\times\ldots\times\ldots\times\ldots\times\ldots}{\ldots\times\ldots}$$

$$= \frac{\ldots!}{\ldots!}$$

Example 11

Find the number of 5-letter permutations that can be formed from the letters in the word SINGAPORE.

Solution

Let us use 5 boxes to represent the 5 letters.

So the number of permutations of 5 letters from the given 9 letters is … x … x … x … x … = …
Using the factorial notation, we have ${}_9P_5$ = … x … x … x … x …

$$= \frac{\ldots\times\ldots\times\ldots\times\ldots\times\ldots\times\ldots}{\ldots\times\ldots}$$

$$= \frac{\ldots!}{\ldots!}$$

Similarly,

$$ {}_nP_r = \frac{\ldots!}{(\ldots-\ldots)!}$$

In general, the number of permutations of r objects from n different objects is

$$P_r^n = {}_nP_r = \frac{\ldots!}{(\ldots - \ldots)!}$$

Example 12

A club has four officials : president, vice-president, secretary, and treasurer. If a member cannot hold more than one office, in how many ways can the officials be elected if the club has :

a. 12 members *b. 16 members*

Solution

a.

b.

C.2 Permutations With Restrictions

Example 13

Find the total number of different permutations of all the letters of the word PELUANG. Find the number of these permutations in which

a. the letters of A and N are together

b. the letters of A and N are not together

Solution

The number of permutations of the 7 different letters = … ! = …

a. To ensure that A and N are together, we gather them together to form one block. Within this block, the 2 letters can be permuted in …! ways.

b. The number of permutations in which A and N are not together
= total number of permutations without restrictions – the number of permutations with A and N together
= … – …
= …

Example 14

4 boys and 5 girls are to form a line. In how many ways can this line be done? Find also, the number of permutations in which

a. the first two are girls,

b. the first is a boy and the last is a girl,

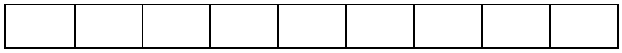
c. the boys are together,

d. no two girls stand next to each other.

Solution

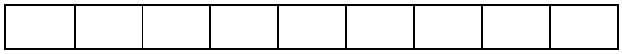
Without any restriction, the 9 people can be permuted in … ! = …….. ways.

a. If the 1st place must go to a girl, it can be filled by any of the … girls. The 2nd place can be filled by any of the other … girls. Thus far, we have



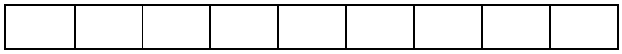
Hence by the basic counting principles, the number of such permutations = … x … x …! = ……..

b. The 1st place can be filled by any of the ... boys. The last place goes to any of the … girls. Thus far, we have



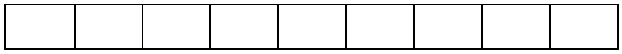
Hence by the basic counting principles, the number of such permutations = … x … x …! = ……..

c. The boys are together



Hence by the basic counting principles, the number of such permutations = …! x …! = ……..

d. No two girls stand next to each other



Hence by the basic counting principles, the number of such permutations = …! x …! = ……..

Exercise 2

1. Find the total number of different permutations of all the letters of the word
   a. TABUNG b. SECONDARY

2. There are five finalists at Dempo Speech Contest. In how many ways can they be arranged to give their speeches?

3. Without using a calculator, evaluate and simplify:
   a. $\frac{9!}{7!}$ b. $\frac{11!}{8!3!}$ c. $\frac{(n-3)!}{n!}$ d. $\frac{(n!)^2}{(n-1)!(n-2)!}$
   e. ${}_7P_3$ f. ${}_{10}P_2$ g. ${}_6P_6$

4. In a Mathematics class with 30 students, the teacher wants 2 different ways to present the solutions to problems 3 and 5 on the board. In how many ways can be teacher assign the problems?

5. A shelf will hold only even number books. Given that 11 different books are available, find the number of different arrangements that can be made to fill the shelf.

6. Four students go for dinner and order a hamburger, a fish burger, a cheese burger and a beef burger (one burger for each). When the waitress returns with the food, she forgets which student orders which item and simply places a burger before each students. In how many ways can the waitress do this?
7. 5 actors and 8 actresses are available for a play which requires 3 male roles and 4 female roles. Find the number of different possible cast lists?

8. In how many ways can 3 boys and 2 girls line up for a group picture? In how many ways can they line up if a boy is to be at each end?

9. Prove that: ${}_{n+1}P_r - {}_nP_r = r.{}_nP_{r-1}$

10. If ${}_{2n}P_3 = 84n$ then find ${}_{n+1}P_2$

C.3 Permutation from n objects with a_1, a_2, a_3, … same objects

Example 15

Find the total number of different permutations of all the letters of the word:
a. ADA
b. SAYA
c. NANA
d. AGAMA

Solution

a. We can list: …..... , …..... , ……

So the number of permutation of word ADA = …..= $\frac{\ldots!}{\ldots!}$

b. We can list: ……. , …….. , …….. , ……. , …….. , …….. , ……. , …….. , …….. , ……. , …….. , ……. So the number of permutation of word SAYA = ….. = $\frac{\ldots!}{\ldots!}$

c. We can list: ……..……..……..……..……..……..……..……..……..……..……..……..……

So the number of permutation of word NANA = ….. = $\frac{\ldots!}{\ldots!\ldots!}$

d. We can list: ……..……..……..……..……..……..……..……..……..……..……..……..……

So the number of permutation of word AGAMA = ….. = $\frac{\ldots!}{\ldots!}$

In general, the number of permutations of n objects with a_1 , a_2 , a_3 , … same object

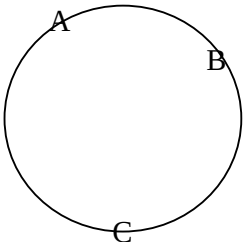
$$\frac{\ldots!}{\ldots!\ldots!\ldots!\ldots \ldots!}$$

C.4 Circular Permutations

Example 16

There are 3 persons (A,B,C) who are sitting in circle. Find the total number of different permutations of them.

Solution



The arrangements are :

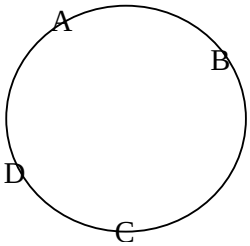
……. , …….

The total number of the cyclical permutation = ….. = ….!

Example 17

There are 4 persons (A,B,C,D) who are sitting in circle. Find the total number of different permutations of them.

Solution



The arrangements are :

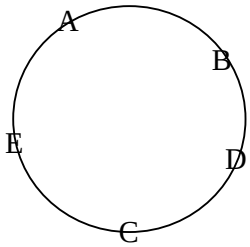
……. , ……. , ……. , …….

The total number of the cyclical permutation = ….. = ….!

Example 18

There are 5 persons (A,B,C,D,E) who are sitting in circle. Find the total number of different permutations of them.

Solution



The arrangements are :

……
……
……
……

The total number of the cyclical permutation = ….. = ….!

In general, the number of circular permutations of n objects: (n-1)!

Exercise 3

1. Find the total number of different permutations of all the letters of the word
   a. SARJANA b. MATEMATIKA
   b. SANTOALBERTUS d. KOMPETENSI

2. How many numbers between 2000 and 5000 can be made from the digits 1, 2, 4, 5, 7, and 8 if each digit is used only once?

3. Each of 7 children, in turn throws a ball once at a target. Calculate the number of ways the children can be arranged in order to take the throws.
   Given that 3 of the children are girls and 4 are boys, calculate the number of ways the children can be arranged in order that
   a. successive throws are made by boys and girls alternately,
   b. a girl taken the first throw and a boy takes the last throw.

4. In a Mathematics class with 30 students, the teacher wants 2 different students to present the solutions to problems 3 and 5 on the board. In how many ways can be the teacher assign the problems?

5. Ami, Beni, Cori, Dani, and Eri went to Dempo concert. How many arrangements were possible when they sit in five adjacent seats if
   a. Eri insists on sitting next to Cori? b. Beni refuses to sit next to Dani?

6. Calculate the number of arrangements of the letters of the world PELUANG if
   a. all the consonants are together, b. no two consonant are together,
   c. each arrangement begins with a consonant and ends with a vowel.

7. Calculate the total number of different permutations of all the letters **A**, **B**, **C**, **D**, **E**, **F** when
   a. there are no restrictions, b. the letters A and B are to be adjacent to one together,
   c. the first letter is **A**, **B**, **C** and the last letter is **D**, **E** or **F**.

8. Twelve jewels of different colour will be arranged to form a clock. Find the number of possibilities.

9. Five books: ABCDE will be arranged on a shelf under the condition: the books B and E are always in adjacent. Find the number of those possibilities.

10. How many phone numbers are there that contain 5 different digits, where the first digits are not zero?

11. 9 different books are to be arranged on a book-shelf. 4 of these books were written by Mira W, 2 by Ayu Utami, and 3 by Remy Sylado. How many possible permutations are there if
    a. the books by Remy must be next to each other?
    b. the books by Ayu are separated from each other?
    c. the books by Remy are separated from each other?

12. At Dempo art exhibition 7 paintings are hung in a row along one wall. Find the number of possible arrangements. Given that 3 paintings are made by the same artist, find the number of arrangements in which
    a. these 3 paintings are hung side by side,
    b. any one of these paintings is hung at the beginning of the row but neither of the other 2 is hung at the end of the row.