Name_____ Date: _____

Introduction to Trigonometry

Today we are going to start looking at trigonometric, or "trig" functions. There are buttons for each of these functions on your calculator and you have probably noticed them before. Sin, Cos, and Tan are the buttons on your calculator that stand for Sine, Cosine, and Tangent.

Each of the functions represents a fraction that you can write using the sides of the triangle. Before you can write the fraction, you need to figure out which sides of the triangle you need to use. This brings us to what SOHCAHTOA stands for:

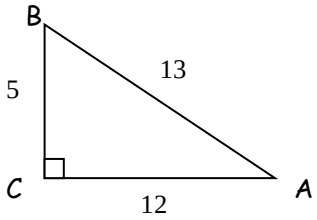
Sin **O**pposite **H**ypotenuse **C**os **A**djacent **H**ypotenuse **T**an **O**pposite **A**djacent

So how do we use this? SOHCAHTOA tells you which sides to use in relation to the angle you are looking at. There are a few steps to doing these problems.

1. Mark the angle you are looking at.
2. Label the sides *in relation to* the angle you are looking at. (opposite, adjacent, hypotenuse)
3. Circle the sides you are supposed to use to make that trig function (use SOHCAHTOA) to help you.
4. Decide which side goes on top of the fraction (numerator) and which goes on bottom of the fraction (denominator).
5. Write the fraction.

Here are a few examples:

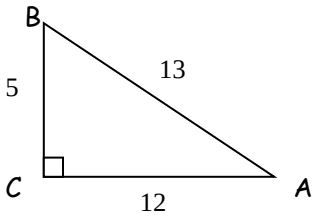
EX 1 Find sin A.



1. Mark angle A.
2. Label the sides in relation to angle A (opp, adj, hyp)
3. Circle the sides that we use for sin (opp, hyp)
4. Decide which side goes on top of the fraction (opp)
5. Write the fraction.

Sin A = _____

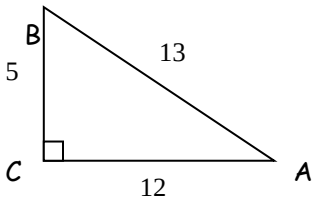
EX 2 Find cos B



1. Mark angle B.
2. Label the sides in relation to angle B (opp, adj, hyp)
3. Circle the sides that we use for cos (adj, hyp)
4. Decide which side goes on top of the fraction (adj)
5. Write the fraction.

Cos B = _____

EX 3 Find tan A



1. Mark angle A.
2. Label the sides in relation to angle A (opp, adj, hyp)
3. Circle the sides that we use for tan (opp, adj)
4. Decide which side goes on top of the fraction (opp)
5. Write the fraction.

Tan A = _____

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Trigonometry with the Calculator

You have learned what sin, cos, and tan mean. Each is a fraction you can write using the sides of a right triangle. But what if you don't know one of those sides? That is where the calculator can help you.

We are still going to use **SOHCAHTOA** to help us. But we need to practice using the calculator before we get to working any problems.

First you need to check that your calculator is in the right mode. It should be in *degrees* mode. Try entering Sin 30°. If you get 0.5 you are in the right mode. Otherwise you need to change it.

After you are sure that you are in degrees mode we are ready to start. Your calculator has information stored into it for every possible angle measure. Enter each of these into your calculator:

1. cos 45° _____ 2. sin 36° _____ 3. tan 18° _____ 4. sin 18° _____

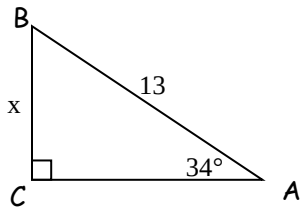
We could keep entering values all day and your calculator would know the answer. (But don't worry, we won't)

Now we will do a couple of examples of how to use the calculator to solve for a missing side. Here are the steps you need to use:

1. Mark the angle that you are going to use for this problem (usually the one that you are given – careful, **don't** use the right angle!)
2. Label the sides *in relation to* the angle that you are going to use (opposite, adjacent, hypotenuse)
3. Circle the sides you are going to use to make a trig function.
4. Decide which trig function you can make with those sides, either sin, cos, or tan.
5. Write the equation, using a variable for the missing side.
6. Solve the equation for the missing side. (using algebra steps)

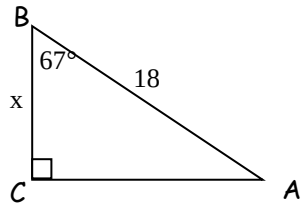
Here are a few examples:

EX 1 Find x.



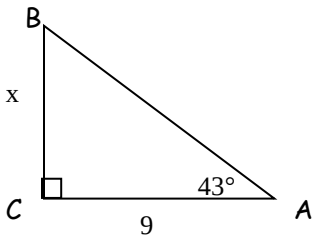
1. Mark angle A (since it is the one that we have a measure for)
2. Label the other two sides *in relation to* angle A (opp, hyp)
3. Circle these sides.
4. Decide which trig function you can make with **O**pp, **H**yp (sin)
5. Write the equation, using the variable x for the missing side.
6. Solve the equation for x, using algebra.

EX 2 Find x.



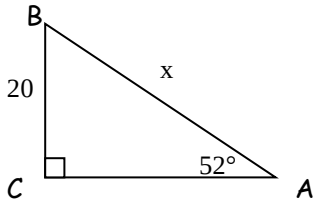
1. Mark angle B (since it is the one that we have a measure for)
2. Label the other two sides *in relation to* angle B (adj, hyp)
3. Circle these sides.
4. Decide which trig function you can make with **A**dj, **H**yp (cos)
5. Write the equation, using the variable x for the missing side.
6. Solve the equation for x, using algebra.

EX 3 Find x.



1. Mark angle A (since it is the one that we have a measure for)
2. Label the other two sides *in relation to* angle A (opp, adj)
3. Circle these sides.
4. Decide which trig function you can make with **O**pp, **A**dj (tan)
5. Write the equation, using the varialbe x for the missing side.
6. Solve the equaiton for x, using algebra.

EX 4 Find x. (this one is a little different!)



1. Mark angle A (since it is the one we have a measure for)
2. Label the other two sides *in relation to* angle A (opp, hyp)
3. Circle these sides.
4. Decide which trig function you can make with **O**pp, **H**yp (sin)
5. Write the equation, using the variable x for the missing side. Be careful here, this is different than the others. Where does x go?
6. Solve the equation for x, using algebra.

Name_____ Date: _____

How to find a Missing Angle using Trig

So far we have just used trig to find missing measures of sides. We can also use it to find missing angles when we know two of the sides. You are going to use your calculator a little differently to do this.

When we know the sides, but don't use the angle we have to tell our calculator that we want to know an angle measure. The way we do this is by pressing 2nd, then either sin, cos, or tan. Then the calculator tells us the degree measure. Here is how it works: (round to the nearest degree)

1. sin A = 0.4226 A = _____ 2. cos B = 0.6691 B = _____

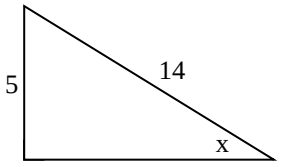
3. tan R = 0.2679 R = _____ 4. sin Z = 0.8290 Z = _____

There are steps we need to use when we are looking for the missing angle measure in a problem.

1. Mark the angle that you are looking for.
2. Label the given sides *in relation to* the angle that you need to find.
3. Decide which trig function (sin, cos, or tan) you can write with the sides that you have been given.
4. Write out the trig function.
5. Use the 2nd sin, cos, or tan to tell you the missing angle.

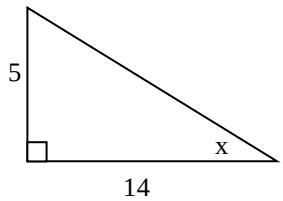
Here are a few examples.

EX 1 Find the missing angle, "x"



1. Mark the angle that we are looking for, x.
2. Label the given sides *in relation to* x. (opp, hyp)
3. Decide which trig function goes with **O**pp, **H**yp (sin)
4. Write out the trig function.
5. Use your calculator to tell you the degree measure.

EX 2 Find the missing angle, "x"



1. Mark the angle that we are looking for, x.
2. Label the given sides *in relation to* x. (opp, adj)
3. Decide which trig function goes with **O**pp, **A**dj (tan)
4. Write out the trig function.
5. Use your calculator to tell you the degree measure.

Geometry Name_____ Date: _____

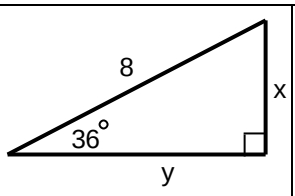
Worksheet – Trig Ratios in Right Triangles

For each of the following, write the equation to find the missing value. Then rewrite the equation that you will enter in your calculator. Round your final answer to the nearest tenth.

1.

$x \approx$ _____

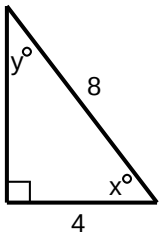
$y \approx$ _____



2.

$x \approx$ _____

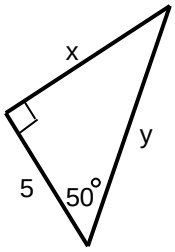
$y \approx$ _____



3.

$x \approx$ _____

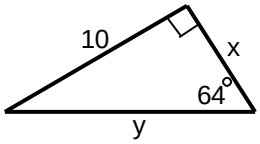
$y \approx$ _____



4.

$x \approx$ _____

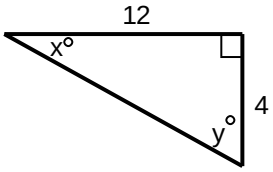
$y \approx$ _____



5.

$x \approx$ _____

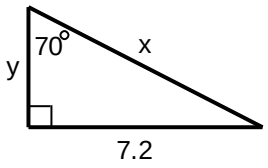
$y \approx$ _____



6.

$x \approx$ _____

$y \approx$ _____

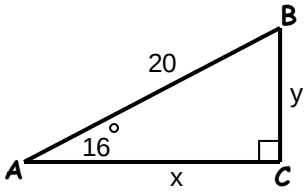


7.

$x \approx$ _____

$y \approx$ _____

$m\angle B =$ _____

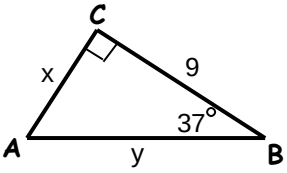


8.

$x \approx$ _____

$y \approx$ _____

$m\angle A =$ _____



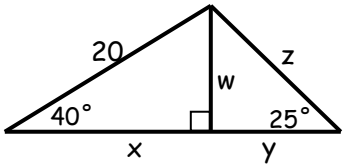
9.

$w \approx$ _____

$x \approx$ _____

$y \approx$ _____

$z \approx$ _____

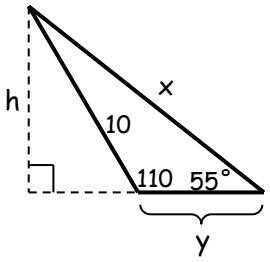


10.

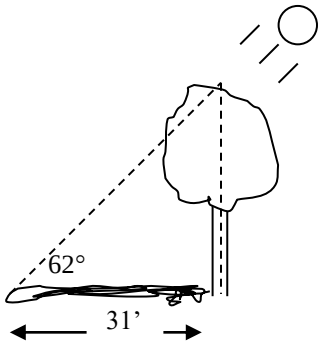
$h \approx$ _____

$x \approx$ _____

$y \approx$ _____



11. How tall is the tree?



12. A man who is 6 feet tall is flying a kite. The kite string is 75 feet long. If the angle that the kite string makes with the line horizontal to the ground is 35°, how far above the ground is the kite?

13. A ladder 14 feet long rests against the side of a building. The base of the ladder rests on level ground 2 feet from the side of the building. What angle does the ladder form with the ground?

14. A 24-foot ladder leaning against a building forms an 18° angle with the side of the building. How far is the base of the ladder from the base of the building?

15. A road rises 10 feet for every 400 feet along the pavement (not the horizontal). What is the measurement of the angle the road forms with the horizontal?

16. A 32-foot ladder leaning against a building touches the side of the building 26 feet above the ground. What is the measurement of the angle formed by the ladder and the ground?

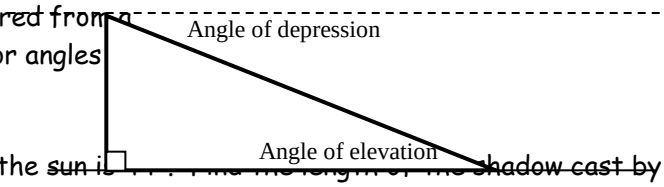
17. The directions for the use of a ladder recommend that for maximum safety, the ladder should be placed against a wall at a 75° angle with the ground. If the ladder is 14 feet long, how far from the wall should the base of the ladder be placed?

18. A kite is held by a taut string pegged to the ground. The string is 40 feet long and makes a 33° angle with the ground. Supposing that the ground is level, find the vertical distance from the ground to the kite.

19. A wire anchored to the ground braces a 17-foot pole. The wire is 20 feet long and is attached to the pole 2 feet from the top of the pole. What angle does the wire make with the ground?

20. A jet airplane begins a steady climb of 15° and flies for two ground miles. What was its change in altitude?

Note that angles of depression and elevation are measured from a horizontal line. Also, because they are alternate interior angles formed by parallel lines they are congruent.



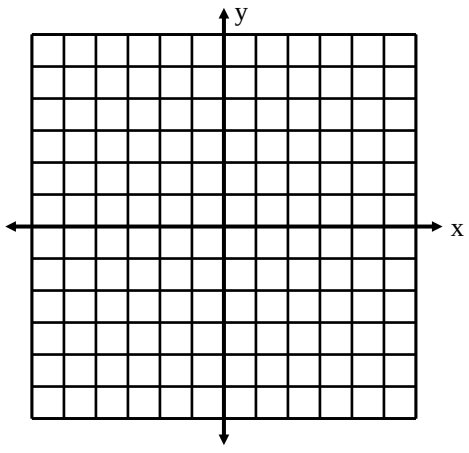
Ex.1 At a certain time of day the angle of elevation of the sun is 71°. Find the length of the shadow cast by a building 30 meters high.

Ex. 2 The top of a lighthouse is 120 meters above sea level. The angle of depression from the top of the lighthouse to the ship is 23°. How far is the ship from the foot of the lighthouse?

Ex. 3 A lighthouse is 100 feet tall. The angle of depression from the top of the lighthouse to one boat is 24°. The angle of depression to another boat is 31°. How far apart are the boats?

To the nearest tenth, find the measure of the acute angle that the line forms with a horizontal line.

$y = \frac{2}{3}x + 1$



Geometry Worksheet Name_____ Date:_____

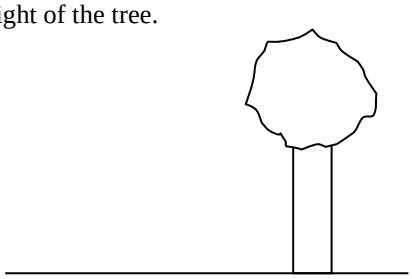
(Angles of Elevation & Depression)

Draw a picture, write a trig ratio equation, rewrite the equation so that it is calculator ready and then solve each problem. Round measures of segments to the nearest tenth and measures of angles to the nearest degree.

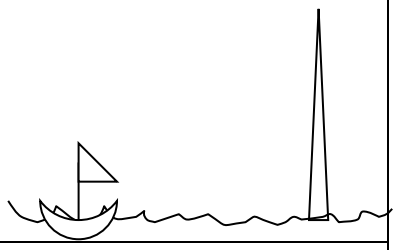
_____1. A 20-foot ladder leans against a wall so that the base of the ladder is 8 feet from the base of the building. What is the ladder's angle of elevation?

_____2. A 50-meter vertical tower is braced with a cable secured at the top of the tower and tied 30 meters from the base. What is the angle of depression from the top of the tower to the point on the ground where the cable is tied?

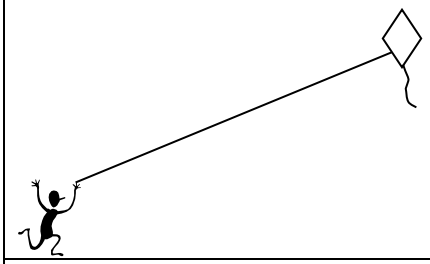
_____3. At a point on the ground 50 feet from the foot of a tree, the angle of elevation to the top of the tree is 53°. Find the height of the tree.



_____4. From the top of a lighthouse 210 feet high, the angle of depression of a boat is 27°. Find the distance from the boat to the foot of the lighthouse. The lighthouse was built at sea level.

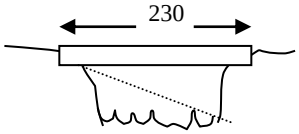


_____5. Richard is flying a kite. The kite string has an angle of elevation of 57°. If Richard is standing 100 feet from the point on the ground directly below the kite, find the length of the kite string.



_____6. An airplane rises vertically 1000 feet over a horizontal distance of 5280 feet. What is the angle of elevation of the airplane's path?

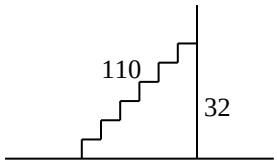
_____7. A person at one end of a 230-foot bridge spots the river's edge directly below the opposite end of the bridge and finds the angle of depression to be 57°. How far below the bridge is the river?



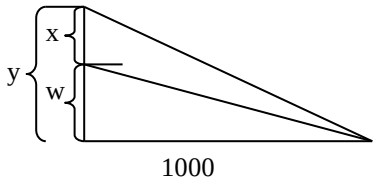
_____8. The angle of elevation from a car to a tower is 32°. The tower is 150 ft. tall. How far is the car from the tower?

_____9. A radio tower 200 ft. high casts a shadow 75 ft. long. What is the angle of elevation of the sun?

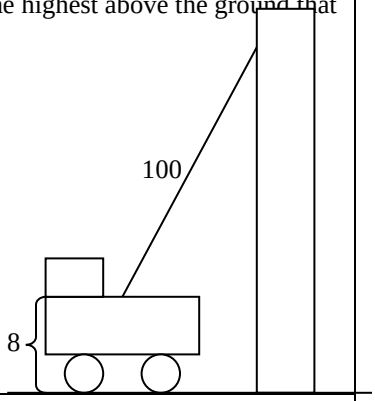
_____10. An escalator from the ground floor to the second floor of a department store is 110 ft long and rises 32 ft. vertically. What is the escalator's angle of elevation?



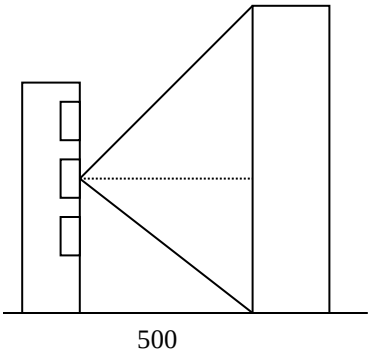
_____11. A rescue team 1000 ft. away from the base of a vertical cliff measures the angle of elevation to the top of the cliff to be 70°. A climber is stranded on a ledge. The angle of elevation from the rescue team to the ledge is 55°. How far is the stranded climber from the top of the cliff? (Hint: Find y and w using trig ratios. Then subtract w from y to find x)



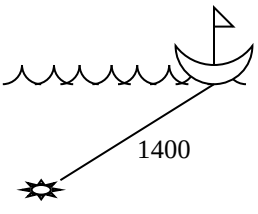
_____12. A ladder on a fire truck has its base 8 ft. above the ground. The maximum length of the ladder is 100 ft. If the ladder's greatest angle of elevation possible is 70°, what is the highest above the ground that it can reach?



_____13. A person in an apartment building sights the top and bottom of an office building 500 ft. away. The angle of elevation for the top of the office building is 23° and the angle of depression for the base of the building is 50°. How tall is the office building?



_____14. Electronic instruments on a treasure-hunting ship detect a large object on the sea floor. The angle of depression is 29°, and the instruments indicate that the direct-line distance between the ship and the object is about 1400 ft. About how far below the surface of the water is the object, and how far must the ship travel to be directly over it?



EXTRA PRACTICE
PYTHAGOREAN THEOREM, SPECIAL RIGHT TRIANGLES, TRIG. RATIOS

1. At a point on the ground 100 ft. from the foot of a flagpole, the angle of elevation of the top of the pole contains a 31 degree angle. Find the height of the flagpole to the nearest foot.

2. Find the length of the side of a square whose diagonal is 6.

3. From the top of a lighthouse 190 ft. high, the angle of depression of a boat out at sea is 34 degrees. Find to the nearest foot, the distance from the boat to the foot of the lighthouse.

4. The congruent sides of an isosceles triangle are each 15 in. and the base is 24 in. Find the length of the altitude drawn to the base.

5. If cos A = sin 30°, then angle A measures how many degrees?

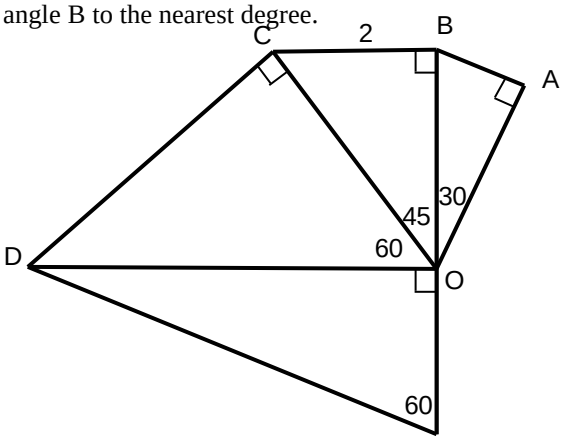
6. Find the length of the diagonal of a square whose side is 6 in. in length.

7. Find to the nearest degree the measure of the angle of elevation of the sun if a post 5 ft. high casts a shadow 10 ft. long.

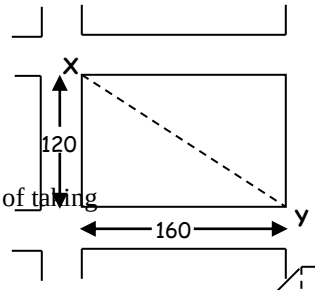
8. The lengths of the bases of an isosceles trapezoid are 8 and 14 and each of the bases angles measures 45 degrees. Find the length of the altitude of the trapezoid and the length of the legs.

9. In triangle ABC, angle C is a right angle, AC = 5, BC = 12.
   a) Find AB.
   b) Find the tan B.
   c) Find sin B.
   d) Find cos B.
   e) Find the measure of angle B to the nearest degree.

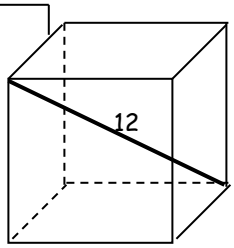
10. AO = _____
    AB = _____
    OB = _____
    OC = _____
    OD = _____
    CD = _____
    DE = _____
    OE = _____



11. How many feet of walking would a person save by cutting across the vacant lot instead of taking the sidewalk around the outside edge?



12. How many inches long must each side of a cubical box be if the distance from one corner is 12 in.? Answer with an expression in simplest form.



13. At a time of day when the sun can be sighted at an angle of 60° above the horizon, a flagpole casts a shadow that is 21 ft long. How tall is the flagpole?

14. The perimeter of a square is 72. What is the length of the diagonal of the square?

15. Find the length of the altitude of an equilateral triangle with perimeter 48.

16. A rectangular box has a square base the area of which is 64 cm^2. The height of the box is 12 cm. Find the length of the interior diagonal of the box.

17. Find the slant height of a regular square pyramid if the altitude is 12 and one of the sides of the square base is 10.

18. A decorator wants the sides of a rectangular picture frame to be in the ratio 7 to 24. If the diagonal is 100 cm. long, what should the lengths of the sides be?

19. A flagpole is at the top of a building. Four hundred feet from the base of the building, the angle of elevation to the top of the pole is 22°, and the angle of elevation to the bottom of the pole is 20°. Sketch a figure. To the nearest foot, find the length of the flagpole.

20. The dimensions of a rectangular solid are in the ratio 3:4:5. If the interior diagonal is $200\sqrt{2}$, find the three dimensions.

21. Terry drove 5 miles east, 7 miles north, 6 miles east, 2 miles south, and 1 mile east. How far is he from his starting point?

Answers:

1. 60 feet
2. $3\sqrt{2}$
3. 282 feet
4. 9 inches
5. 60°
6. $6\sqrt{2}$ inches
7. 27°
8. altitude = 3; leg = $3\sqrt{2}$
9. (a) 13; (b) 5/12; (c) 5/13; (d) 12/13; (e) 23°
10. AO = $\sqrt{3}$; AB = 1; OB = 2; OC = $2\sqrt{2}$; OD = $4\sqrt{2}$; CD = $2\sqrt{6}$; DE = $\frac{8\sqrt{6}}{3}$; OE = $\frac{4\sqrt{6}}{3}$
11. 80 feet
12. $4\sqrt{3}$ inches
13. $21\sqrt{3}$ feet
14. $18\sqrt{2}$
15. $8\sqrt{3}$
16. $4\sqrt{17}$ cm.
17. slant height = 13
18. 28 cm and 96 cm
19. 16.022 or about 16 feet
20. 120, 160, and 200
21. 13 miles